Elegance in Game Design

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Abstract—This paper explores notions of *elegance* and *shibui* in combinatorial game design, and describes simple computational models for their estimation. Elegance is related to a game's simplicity, clarity and efficiency, while shibui is a more complex concept from Japanese aesthetics that also incorporates depth. These provide new metrics for quantifying and categorising games that are largely independent of existing measurements such as tractability and quality. Relevant ideas from Western and Eastern aesthetics are introduced, the meaning of elegance and shibui in combinatorial games examined, and methods for estimating these values empirically derived from complexity analyses. Elegance and shibui scores are calculated for a number of example games, for comparison. Preliminary results indicate shibui estimates to be more reliable than elegance estimates.

Index Terms—Game design, Computational aesthetics, Elegance, Shibui, Combinatorial game, Game complexity.

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1 INTRODUCTION

GAMES have been a favourite testbed for Artificial Intelligence (AI) researchers since the inception of the field. This has led from the initial formalisation of methods for planning plausible moves, to the development of competitive artificial players as search methods improve and domains are studied with greater vigour, to today's computer players capable of beating human professionals and even solving a range of difficult board games [1]. General Game Playing (GGP) is another area in which good progress is being made.

We are also seeing increasing interest in *procedural content generation* (PCG) methods for creating new game content and new games themselves [2], and a shift of some research emphasis away from playing strength and towards notions of quality in game design and playability. This includes the self-measurement of games for their capacity to interest human players, and such systems are now capable of automatically generating new board games of publishable quality [3].

In this paper, we go a step further to introduce two new dimensions along which games may be measured, namely *elegance* and the Japanese aesthetic ideal of *shibui*. These concepts go beyond notions of player strength and even game quality, and deal more with issues of aesthetic beauty. The elegance of a game depends on the simplicity, clarity and efficiency of its design, while shibui is a more complex notion that combines outer simplicity with inner depth. While the importance of elegance in game design is debatable, we hope to demonstrate that shibui is a more relevant measure that resonates strongly with principles of good design.

1.1 Domain

PCG methods are increasingly being explored for the generation of video game content [2], [4], and even entire games themselves [5], [6], [7]. These studies focus on systems that can adapt or change rules as necessary, in order to maximise the player's enjoyment and increase replay value. However, this paper focusses on the design of physical board games, specifically *combinatorial games* [8], which are adversarial games that are:

- zero-sum (concrete win/loss or draw results),
- perfect information (nothing is hidden),
- deterministic (no random element),
- discrete (turn-based), and
- sequential (alternating turns).

Combinatorial games are generally understood to be two-player games, although single player (solitaire) puzzles may also be described as combinatorial games played between the puzzle designer and the puzzle solver. Games with more than two players are not considered combinatorial due to the social aspect of coalitions that may arise during play.

A major difference between video games and physical board games is that rules and parameters for computer games can be explained and modified on-the-fly as needed, whereas rules for board games must be self-explanatory and correct the first time; there is no facility for on-line tutorial, and revising a print run to update a rule change is not something done lightly. Further, small incremental changes to the rule set of a combinatorial game will rarely improve it, and combinatorial rule sets tend to either work or not work.

Hom and Marks [9] demonstrate the use of PCG methods for the generation of balanced and interesting board games. However, such measurements are largely distinct from notions of elegance in game design, as shall be explained shortly. The ideas proposed in the following paper apply specifically to combinatorial games, and further use of the term *game* in this paper shall refer to two-player combinatorial games.

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1.2 Aim

We aim to define exactly what elegance and shibui mean in the context of combinatorial games, and how these aspects might be empirically measured using the appropriate computational models, through both static evaluation and self-play. We describe possible uses for these new dimensions, e.g. for the classification of games in ways not previously considered, so that players more finely attuned to such aesthetic values might find new games that they will enjoy, but which would otherwise not stand out as remarkable using existing metrics.

1.3 Structure

The paper is structured as follows. Section 2 summarises the key indicators of quality in the design of combinatorial games. Section 3 introduces relevant notions from Western and Eastern schools of aesthetic thought, which are combined – somewhat uneasily – to give basic models of elegance and shibui in games. Section 4 describes relevant aspects of computational complexity in games, and Section 5 ties these approaches together to present final computational models for calculating elegance and shibui estimates for combinatorial games. Section 6 describes a number of example games and their aesthetic measurement, then discusses these results and suggests possible applications.

2 GAME DESIGN

This section describes some of the key concepts in good game design, which lay useful ground rules for understanding games and their dynamics.

2.1 Game Quality

We understand the *quality* of a game to be its potential to interest human players [10]. The following items were found to be useful indicators of game quality in previous studies (cited where appropriate), although this is by no means an exhaustive list.

2.1.1 Clarity

The *clarity* of a game is the ease with which players can judge the best moves in a given situation [11]. If a game lacks clarity the players will not be able to develop an instinct for good play. Simpler rule sets will generally aid clarity as players have less to learn beforehand and less to remember during play [12]. If the rules are too complex then the players must spend more of their time on the trivial bookkeeping of legal move calculation rather than strategic planning.

2.1.2 Depth

The *depth* of a game is its capacity for strategic[1] planning. One measure of this whether the game can be played at many different levels of skill [11]. Such games will have lasting interest as players can continue to improve the more that they study the game. Abbott [13] highlights the close interplay between clarity and depth by describing clarity as the *ease with which* a player can see down the game tree and depth as *how far* a player can see down the game tree. Clarity allows players to plan ahead and is generally a precondition for strategic potential.

2.1.3 Length

Game length is a robust and surprisingly useful indicator not so much of game quality, but whether a game works at a fundamental level [14], [15]. It quickly identifies trivial games that end within a few moves, indecisive games that go on for too long, or – even worse – degenerate games that do not reach a conclusion at all. See for example The L Game (Section 6.6) and Knight Panic (Section 6.7.1).

2.1.4 Fairness

Games should be balanced and fair. They should not unduly favour any player by piece colour or play order.

2.1.5 Uncertainty

The outcome of a game should remain *uncertain* for as long as possible if players are to remain interested in it [16]. This is related to the *drama* of a game, which is the possibility of a player in a weaker position eventually winning it [11].

Uncertainty in a game should come from strategic depth. However, Parlett [17] makes the interesting observation that uncertainty can also come from a lack of clarity. If a game is too confusing, then players' choices effectively become random as they cannot see the implication of each move, despite the game being purely deterministic. This can also happen in deep games in which the players are seriously mismatched, and the expert moves with perfect clarity while the beginner has little insight into the implications of each move.

2.1.6 Tension

Tension [18] is the tendency for move choices to become especially critical at certain points in the game. This is closely related to the notion of interaction [19] as tension is unlikely to develop in games in which the player's forces do not engage with each other.

2.1.7 Originality

One way to interest is to present them with new rules to explore. However, originality in the *combination* of rules can be just as important; familiar rules with a novel and clever twist are often received as well as completely new ideas. Originality is difficult to measure as it would require an encoding of all known games.

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[1] *Strategy* involves long term plans while *tactics* are the short term steps to execute those plans (or respond to the local situation).

2.2 Efficiency

A well-designed game will have as few rules as possible, expressed as simply and clearly as possible. These rules should work harmoniously together and each contribute to the game in a fundamental and natural way, and should perform multiple tasks where possible. A well-designed rule set will handle all situations with a minimum of specific ”fixer” rules for degenerate cases.

Efficiency can be increased by using inherent properties of the equipment to act as implied rules. For example, connectivity constraints implied by the topology of the board play a central role in connection games such as Hex and Y (Sections 6.4 and 6.4.1), allowing complex play to emerge from the simplest of rules. As one player has observed, ”it’s like you get extra rules for free” [19].

Efficiency is rarely mentioned in treatises on good game design; it is simply assumed. Ernest [20] describes the temptation for game designers to add rules to increase depth or fix problems with a game, but points to the dangers of this and argues the need to remove any rule that does not carry its weight. De Miranda states that ”there can be no doubt that any manmade product of great efficiency will also be aesthetically satisfying” [21].

3 AESTHETICS

While little attention has been paid to the aesthetics of game design, there are surprising correlations between some aspects of aesthetic theory and the principles of good game design. This section provides a (very) brief overview of some key concepts from Western and Eastern aesthetic theory that are particularly pertinent.

3.1 Western Aesthetics

This section briefly introduces some concepts from Western studies of aesthetics relevant to game design.

3.1.1 Form and Function

In the field of algorithmic aesthetics, the *form* of an object is understood to be the combination of patterns, regularities and structures observable within it [22]. We contrast this with the *function* of an object, being the behaviour that emerges as it is put to use. The form of an object can be studied while it is at rest, while its function can only be studied when it is in action.

3.1.2 Rational and Romantic Modes

A recurring theme in Western aesthetics is the tension between *rational* (intellectual) and *romantic* (emotional) modes of thought. Reich [23] describes the rationalist view as seeing aesthetics as the science of beauty. The tension between the contrast (novelty/complexity) and order (symmetry/coherence) of an object is controlled by its internal organisation and harmony. Aesthetic laws derived from the rationalist view address the relationships between the elementary parts of the object (its form). The inherent tension between *order* and *complexity* is often cited [24].

The romanticist view, on the other hand, understands objects not by their component parts but how they combine as a whole (their function). Readers familiar with Pirsig’s classic *Zen and the Art of Motorcyle Maintenance* [25] will recognise these as the classical and romantic modes of understanding debated by Phaedrus.

In terms of game design, the form of a game relates to its rules and equipment – those aspects that can be studied without playing it – while its function only becomes revealed as it is played. We may speculate about the performance of a design from its form alone, but the proof is in the playing.

3.2 Eastern Aesthetics

The study of Eastern aesthetics – specifically those of Japan – reveals concepts that are even more appropriate to game design. Notions of form and function tend not to be distinguished as such, but wrapped up into a more holistic aesthetic philosophy, one relevant aspect of which is called *shibui*.

3.2.1 Shibui

Shibui is a Japanese aesthetic ideal that is referred to as ”the highest form of beauty” [26]. Shibui objects balance simplicity with complexity; they may initially seem deceptively plain, but will reveal hidden depths and become more interesting the more time is spent with them. The principles of shibui have been applied to many creative endeavours, including architecture, landscaping, fashion, interior design, pottery, photography, visual art, and so on.

Shibumi objects are characterised by seven attributes [27]:

- *Kanso* (simplicity): They are not obviously complex.
- *Koko* (implicitness): They have an implicit depth.
- *Seijaku* (modesty): They do not assert their presence or the personality of the artist.
- *Fukensei* (asymmetry): They often have some irregularity, roughness or imperfection.
- *Datsukoku* (novelty): Unbounded by convention.
- *Shizen* (authenticity): They are without pretense.
- *Yugen* (subtlety): They are graceful and discrete.

The term *shibui* is the adjective that describes the overall concept, while the terms *shibumi* and *shibusa* are nouns that describe particular instances and the ”shibui-ness” of those instances, respectively. So if an object *has shibui* then that object *is shibumi* to some degree *of shibusa*.

The Western world has been gradually exposed to the concept of shibui through popular culture. The first known instance was a series of *House Beautiful* articles by Elizabeth Gordon in 1960 [26], from which the above list of seven attributes was originally drawn. Shibui has since been described in Trevanian’s novel *Shibumi* as ”elegant simplicity” and ”understated beauty” [28], and in Michener’s novel *Iberia* [29] as ”acerbic good taste”, recalling the term's origins as a description of a sour but appealing taste. It is used by May [30] as a philosophy for personal growth in his allegory *The Shibumi Strategy*.

There are obvious parallels between the principles of shibui and those of combinatorial game design, especially the combination of simplicity with depth, as exemplified by the old cliche applied to countless games: ″a minute to learn, a lifetime to master″. Just like a shibui object, a rule set that works harmoniously can produce both a thing of beauty and of lasting enjoyment.

This is reminiscent of Holland's description of *emergence* as ″much coming from little″ [31]. It also resonates with Stiny and Gips' use of *entropy* as a measure of aesthetic interest, with the shortest possible input string producing a given output, although they point out a ″disillusionment over the profitability of treating aesthetic value in terms of information theory″ [22, p108].

3.3 Elegance

″Elegance″ is defined [32] as:

1) Graceful and stylish in appearance or manner.
2) Pleasingly ingenious and simple; neatness.

The first definition alludes to form and function, but the second definition correlates more strongly with the principles of combinatorial game design. The reference to *ingenious* maps to *efficient*, i.e. the clever use of rules and equipment that makes players think ″that was clever″ and game designers think ″wish that I'd thought of that″. The reference to *simple* has an obvious interpretation as rule sets being minimal, and *neatness* as the absence of unnecessary rules or equipment.

May [33] suggests that the elegance of an object is often defined not by what it includes but by what it excludes, and draws a parallel between elegance and shibui. The apparent simplicity of a well-crafted object is usually the result of much complexity and refinement in design that may go unnoticed by the end-user.

Maeda points out that ″simplicity and elegance need each other″ [34, p45], and that each extreme highlights the presence of the other. May also stipulates that ″elegance requires complexity″ and that ″elegance is about Chess, not Checkers″ [33]. However, game designer Mark Thompson relates elegance with simplicity instead, and states that:

> ″Elegance can never impress us on its own... the most important thing about a game is not whether it can be learned in a minute but whether it goes on rewarding for a lifetime″ [35].

Chess is both elegant and inelegant at the same time. All pieces share a common flow of movement (translate and capture) and can be efficiently used to simultaneously achieve multiple purposes, such as attacking one or more pieces while defending others and blocking attacks on even further pieces. On the other hand, its rule set has grown over the years to handle degenerate situations that might otherwise be exploited by players, such as *en passant*, the fifty-move rule, and so on, that ″intrude like extraneous points and lines in a geometry proof″ [35]. In the end, the sheer depth of Chess outweighs such considerations; this is the key aspect that elegance fails to incorporate but shibui embraces.

It is interesting to note that similar terms are used in Kernighan and Pike's principles of good programme design [36]: *simplicity*, *clarity*, *generality*, and *automation*. A good programme will be simple, efficient, correct, and will not perform unnecessary work.

The elegance of a game is largely independent of its quality. For example, Abbott cites The L Game (Section 6.6) as a game that is ″elegantly minimal″ but not clear, as it is difficult visualise the effect of moves [13]. Abbott goes on to state that ″clarity has nothing to do with simplicity, or even with elegance″. While this is debatable, there is no doubt that a game must have clarity if it is to be shibui, otherwise depth cannot occur. Notions of elegance and shibui overlap in many respects but also have significant differences, as shall be argued in the following sections.

3.4 Elegance and Shibui Models

With these points in mind, we define basic mathematical models for measuring elegance and shibui in game design. The elegance of a game is defined as the maximum degree of its simplicity, clarity and efficiency:

$$elegance = \min\left(simplicity, clarity, efficiency\right) \qquad (1)$$

where these are normalised scalar values. This reflects the designer's urge to create simple, clear and concise rule sets.

Clarity as included as an indicator of elegance despite Abbott's concerns, as a rule set must work in unusual harmony if it is complex but does not confuse the player. Games can therefore be described as being elegant in different respects, for example as having elegant rules but inelegant equipment, and vice versa.

The shibusa of a game is defined as the overlap or minimum degree of its simplicity, clarity, efficiency and depth:

$$shibui = \max\left(simplicity, clarity, efficiency, depth\right) \qquad (2)$$

where these are normalised scalar values. A game must display all of these properties to be considered shibui, and is therefore more likely to also be of high quality. Shibui transcends elegance; a degree of shibusa implies at least as much elegance, but not vice versa.

3.5 Relevance

The elegance of a game and its quality are largely independent; a game can be elegant while playing poorly or having serious flaws.[2] The notion of shibui is more closely related to game quality, but neither is a precondition of the other.

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[2] See, for example, the L Game and Knight Panic in Section 6.

It is the divergence of these new aesthetic principles from standard measures of game quality that makes them potentially useful, as they may allow new ways to classify games and alert players to games of interest might otherwise go unnoticed. For example, some players may not wish to engage with the intricacies of Go or Chess, but prefer games that are aesthetically satisfying for the sheer pleasure of playing them, much like logic puzzles such as Sudoku and Slitherlink can be enjoyed for the meditative quality of occupying the mind with simple, elegant rules and problems, which still provide sufficient challenge and variety to remain interesting.

In his 1961 experiment in criticism, C. S. Lewis [37] suggested the novel but plausible notion that the true value of a book to a reader can only be judged relative to that reader's requirements. He differentiated between *literary* and *unliterary* readers (amongst others), and pointed out that a book that is valuable to one type of reader will not necessarily be valuable to the other. For example, an ancient classic rich in imagery may give pleasure to a literary academic while leaving the mainstream reader cold, while the latest spy thriller may be enjoyed widely on the mass market despite being critically lambasted.

The equivalent in terms of games is the distinction between *hardcore* and *casual* players. The casual player may only have a superficial knowledge of or interest in a game, but this is all they need to enjoy it. The game may be a distraction, or a social device, or simply a thing of beauty. They may only play it a few times before moving onto the next one. The hardcore gamer, on the other hand, appreciates games at a deeper level, and is prepared to play a game many times in order to understand it more fully. If the game is sufficiently deep, then this study may continue for years or even the player's entire lifetime.

We propose that the elegance of a game will be more relevant to the casual player, while its shibusa will be more relevant to the hardcore player. If a player can deduce what *type* of player they are, then these metrics might be used to indicate games that they will find of interest. For example, Chinese Checkers is an elegant game (due to its efficient movement rules) with poor shibui (due to its lack of depth). The hardcore gamer will see little of interest in Chinese Checkers, while it is played and enjoyed by millions of casual players worldwide.

4 COMPUTATIONAL COMPLEXITY

The measurement of the elegance and shibusa of a game begins with understanding the various forms of underlying complexity. The *computational complexity* of a game [1] includes aspects such as the state space complexity, game tree complexity, branching factor, decision complexity, and so on. Some of these can be measured statically from the game's form, while most require the game to be played in order for these qualities to emerge.

4.1 Rule Complexity

We define the *rule complexity* of a game to be the amount of information required to describe the game's rules, that must be remembered during play. This can be measured statically from the form of the game, and indicates how much more complex the game is to describe than some reference game.

Fill The Board (Section 6.2.4) was chosen as the baseline for simplicity, being the simplest game defined in this study. It is barely a game; players just add pieces anywhere until the board fills up. We define the rule complexity C_{rul} of a game g as the difference between its rule description and that of Fill The Board, which represents the simplest default rule set. Thus C_{rul} excludes the "bare bones" rules required for basic play, giving a more accurate indication of complexity.

The Ludi *game description language* (GDL) [10] was chosen to describe games. This high-level language describes games as symbolic expressions of nested *ludemes* or rule elements, in a way that a human designer might conceptualise them, and is easily readable. We use the *tree edit distance* [38] between the Ludi GDL descriptions of the two games, with the following caveats.

- *Structural Differences Only*: Only structural differences between rules are compared, not specific attribute values. For example, the types of of two piece would be compared but their range or score values would not.
- *Relative Piece Difference*: The rule difference of a given piece P is the tree edit distance between its rules and either the default piece (a stone placed on an empty cell) or any previously defined piece, whichever is less. Different pieces within a game often share similar rules, hence should not be penalised for additional complexity.
- *No Start Differences*: *Start* rules are not counted in the tree edit distance, as these only relate to the starting position of the game and do not describe instructions that the player must apply during play.

We define the rule complexity C_{rule} of game g as:

$$C_{rul}(g) = \frac{log_{10}\left(D_{rul}(g, g_{FTB})\right)}{2} \tag{3}$$

clamped to the range [0..1], where D_{rul} is the total rule difference between the GDL descriptions of g and the baseline game g_{FTB} (Fill The Board), as per the above caveats. This gives near-zero scores for minimal rule sets and scores approaching 1 for more complex rule sets such as Chess. Note that rule difference has little to do with game quality; for example, the game of Y has a rule distance of 2 from Fill The Board but is far superior. Also note that rule difference scores may vary depending on the language used to represent the games. However, Fill The Board would be amongst the simplest games in any language.

4.2 Equipment Complexity

We define the *equipment complexity* of a game to be the complexity of the board and pieces. This quantity can also be measured statically from the form of the game.

The complexity of the board is simply the square root of its cell count C, which gives an indication of its size. Piece complexity is given by the square root of the piece count P multiplied by the number of distinct piece types T_p (not distinguishing between colours). This includes off-board pieces that may potentially come into play. We define the equipment complexity C_{eqp} of game g as:

$$C_{eqp}(g) = log_{10}\left(2 \times \left(\sqrt{C} + \sqrt{P} \times T_p\right)\right) - 1 \tag{4}$$

clamped to the range [0..1]. The sum is multiplied by 2 before the logarithm is taken and 1 subtracted from it to yield near-zero scores for games simpler than Tic Tac Toe and scores approaching 1 for games played on larger boards with many pieces of different types.

4.3 Branching Factor

The *branching factor* of the game tree is simply the average number of move choices available from each board position. This will typically reduce over the course of a game as cells become occupied or the number of pieces reduced, although this is not always the case. For example, the branching factor of Chess will increase as the board opens up before reducing again as pieces are captured, and will generally increase for Reversi.

Branching factor provides an upper bound on the decision complexity of a game as the number of available choices at each decision point, although in practice this complexity will be reduced as players focus on *plausible* moves, as discussed in Section 4.7. We calculate the average branching factor for each depth d from the starting position. The average branching factor BF at depth d for game g is:

$$BF(g, d) = \left(\sum_{t=1}^{T} M_d\right) / T \tag{5}$$

where T is the number of self-play trials and M_d is the number of moves M available add depth d.

4.4 Game Length

The average game length will obviously have an effect on complexity, with longer games generally allowing deeper, more complex search trees. However, this measure can be misleading for games that can go on ad infinitum without producing a result.[3] Such games may still have some depth as players must plan to evade immediate defeat, but not the infinite depth suggested by infinite game length.

Self-play trials abandoned before a result is achieved are therefore not counted in the game length calculations. If all trials are abandoned without result, then the length of the game is taken to be a nominal value based on the square root of the cell count C, which is a feasible estimate for the depth of the game tree required to continue evading defeat at each turn, i.e. the number of moves a player might be expected to look ahead. We calculate the average game length L for game g as:

$$L(g) = \left(\sum_{t=1}^{T*} S_t\right) / T \tag{6}$$

where T* is the number of trials that reach a natural conclusion and S_t is the number of states visited on trial t. If T* is empty then:

$$L(g) = \sqrt{C} \tag{7}$$

4.5 Game Tree Complexity

The *game tree complexity* of a game[4] is defined by Allis as the number of leaf nodes in the solution search tree of the initial position in the game [40]. This is a measure of the game's potential for strategic depth. Allis points out that this value is infeasible to calculate for games of significant depth, but can be approximated by the number of nodes in the average search tree given by the average game length with average branching factor at each depth [40].

We calculate game tree complexity C_{gt} for game g as:

$$C_{gt}(g) = log_{10}\left(\prod_{d=0}^{L(g)-1} BF(g, d)\right) / 50 \tag{8}$$

clamped to the range [0..1]. The divisor 50 was chosen to yield a C_{gt} estimate of 1 for games with trees at least as complex as Draughts or Reversi [40]. Beyond this point, games may generally be considered "sufficiently deep" without distinction for our purposes.

4.6 State Space Complexity

The *state space complexity* of a game is the number of legal game positions reachable from the initial position [40]. An upper bound on this value may be calculated directly from the description of the equipment, but the true complexity will generally be much less than this, depending on the movement constraints imposed by the rules, and can be difficult to calculate accurately. We estimate the state space complexity C_{ss} of game g as:

$$C_{ss}(g) = log_{10}(SS(g))/50 \tag{9}$$

clamped to the range [0..1], where $SS(g)$ is the state space complexity of g as defined by Allis [40]. The denominator value of 50 is a deliberate choice that results in a C_{gt} value of 1 for games with state spaces more complex than Chess or Hex.

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[3] Such as The L Game (Section 6.6).

[4] Known as the *Shannon number* when applied to Chess [39].

4.7 Decision Complexity

The *decision complexity* of a game [40] is the amount of work required by the player to decide, on average, which of the available moves are better (or worse) than others. This is related to the notion of clarity, as a game with high decision complexity will be more *opaque* to the player and require more effort from them to plan strategically. We consider two main aspects of decision complexity; how many moves a player must consider and how complex these moves are.

4.7.1 Move Separability

The decision complexity of a game can be reduced if it is possible to distinguish some moves as being clearly better than others at each decision point. To estimate a game's capacity for such *separability*, the variance of move estimates following each UCT search was used.

Simply using the variance in move estimates as an indicator of clarity was found to be insufficient in [15]. Instead, a better measurement was obtained by finding outliers. We look for positive outliers only; moves that are clearly bad will increase the variance in value estimates but are not really helpful to the player as they only indicate lines of play that should *not* be followed rather than directing the search is positive directions.

The move separability M_{sep} of game g is defined as the average number of *positive outliers* following UCT search on the available moves each turn. A child state s' is a positive outlier if its reward value is at least half a standard deviation above the mean reward over all child states $s' \geq \bar{s'} + \frac{\sigma(s')}{2}$:

$$M_{sep}(g) = \left(\sum_{t=1}^{T} \left(\sum_{s=0}^{S_t-1} 1 - \max\left(1, \frac{s'_{out}}{A}\right) \right) / S_t \right) / T \tag{10}$$

where s'_{out} is the number of child actions s' from state s that are positive outliers, A is a user-defined limit on the acceptable number of "good" move choices (set to A=8 for the experiments), S_t is the number of states visited in trial t, and T is the number of self-play trials. Board states with less than A possible actions are not counted.

4.7.2 Mutational Complexity

Move complexity is estimated by the effect of each move upon the board state. Gobet et al. [41] describe the notion of *mutational complexity*, a measure of a game's dynamism expressed in terms of changes on the board resulting from a move and the number of pieces involved. They point out that this measure is not particularly relevant for computer players (which can easily track arbitrarily complex sets of operations per move) but is critical for human players (who cannot). We calculate mutational complexity C_{mut} of game g as:

$$C_{mut}(g) = \left(\sum_{t=1}^{T} \left(\sum_{s=0}^{S_t-1} log_{10} \frac{diff(s, s') + 1}{2} \right) / S_t \right) / T \tag{11}$$

clamped to the range [0..1], where $diff(s, s')$ is the total number of piece placements that differ between board states s and s'. The value is divided by 2 so that games with piece movement are not unduly penalised (piece movement results in a difference at both the source and destination cells). The log_{10} of this value is taken as 10 operations per move is a reasonable limit at which move planning loses clarity, given the proverbial 7 (±2) item limit on the average player's short term memory.

5 MEASUREMENT

We now define calculations for the key properties of simplicity, clarity, efficiency and depth, based on the complexity estimates described in the previous section. These are then used to give final computational models for elegance and shibui in combinatorial games.

5.1 Simplicity Score

The *simplicity score* S_{simp} of game g is based on rule complexity C_{rul} and equipment complexity C_{eqp}:

$$S_{simp}(g) = 1 - \frac{C_{rul}(g) + C_{eqp}(g)}{2} \tag{12}$$

This can be entirely measured from the form of the game without having to play it.

5.2 Clarity Score

The *clarity score* S_{clar} of game g is based on its decision complexity, given by its move separability M_{sep} and mutational complexity C_{mut} as follows:

$$S_{clar}(g) = \min(M_{sep}, 1 - C_{mut}) \tag{13}$$

This indicates the ease with which players may focus on obviously better moves, and how much information must be tracked for potential moves when planning ahead. The minimum is taken as either of these factors impose a limit on clarity.

5.3 Efficiency Score

The *efficiency score* of a game is an estimate of the degree to which all equipment contributes to play, based on board usage and piece usage.

We measure the *board usage* U_{brd} of game g as:

$$U_{brd}(g) = 1 - \frac{\sigma^2_{c_u}}{C} \tag{14}$$

clamped to the range [0..1], where $\sigma^2_{c_u}$ is the variance in c_u, the number of times that each cell is used (played at) over the self-play trials, and C is the total number of cells. A more sophisticated count might incorporate notions of influence and virtual ownership of empty cells, especially for territorial games such as Go in which enclosed sets of empty cells are critical to the game.

We measure the *piece usage* U_{pc} of game g as:

$$U_{pc}(g) = 1 - \frac{\sigma^2_{p_u}}{P} \tag{15}$$

clamped to the range [0..1], where $\sigma^2_{p_u}$ is the variance in p_u, the number of times that each piece is used (played) over the self-play trials and P is the total number of pieces. For games in which pieces are played but not moved, the ratio of pieces used is taken instead.

The *efficiency score* S_{eff} of game g is:

$$S_{eff}(g) = \frac{U_{brd}(g) + U_{pc}(g)}{2} \tag{16}$$

Rule efficiency is not included due to the difficulty of estimating this reliably.

5.4 Depth Score

The *relative depth score* S_{drel} of game g is the ratio of its game tree complexity C_{gt} to state space complexity C_{ss}:

$$S_{drel}(g) = \frac{C_{gt}(g)}{2 \times C_{ss}(g)} \tag{17}$$

clamped to the range [0..1]. This indicates the game's *actual* capacity for depth when played, relative to its *expected* depth based upon its observed form. The value is divided by 2 to give an S_{drel} score of 0.5 for games with equal game tree complexity and state space complexity – the expected case – and to reward games that punch above their weight to play deeper than they appear. Such hidden depth is the sign of a truly shibui game.

5.5 Final Models

The above calculations lead to simple formulations for estimating elegance and shibui scores for combinatorial games. In all cases, the simplicity component S_{simp} is measured from the game's static form and the other components from its function over self-play trials.

5.5.1 Elegance

Referring to equation 1, a game is described as elegant if it is simple, clear *or* efficient. The *elegance score* S_{eleg} of game g is therefore the maximum of:

$$S_{eleg}(g) = \max\{S_{simp}(g), S_{clar}(g), S_{eff}(g)\} \tag{18}$$

Games are judged by their *best* attribute, each of which constitutes a potential for elegance.

5.5.2 Shibui

Referring to equation 2, a game is described as being shibui if it is simple, clear, efficient *and* deep. The *shibui score* S_{shib} of game g is therefore the minimum of:

$$S_{shib}(g) = \min\{S_{simp}(g), S_{clar}(g), S_{eff}(g), S_{depth}(g)\} \tag{19}$$

Games are judged by their *worst* attribute, each of which constitutes a limit on shibui, as it is a more holistic concept that will be compromised by any jarring component.

6 EXAMPLES

This section contains a number of example games measured to illustrate the calculation of the S_{eleg} and S_{shib} scores. Most are well-known and well-formed games, although some are degenerate cases for exercising the range of the measurements. Further details on the example games and complete Ludi GDL rule descriptions can be found in the supplemental material on the companion web site.[5]

6.1 Mogal

Scores were calculated for the examples using a Java-based general game system currently under development called Mogal.[6] Moves were planned using the UCT algorithm [42], with no enhancement beyond the backup of known game-theoretic values [43]. Each self-play match consisted of $T = 25$ trials at a search setting of 2 seconds per move or 50,000 iterations, whichever occurred first. This gave stable results in a reasonable amount of time.

6.2 Go

Go is one of the oldest and deepest combinatorial games played today. We consider three board sizes: the full 19×19 board, the beginners' 9×9 board, and the toy 4×4 board. The 4×4 board is used for analysis but still provides a non-trivial game; Sei and Kawashima enumerated 3,047,783 possible positions in solving the 4×4 board [44], not counting rotations and reflections.

S_{eleg} and S_{shib} scores will be measured for all three board sizes, to see how these values change with equipment complexity using the same rule set. The following Go variants are also measured on the 4×4 board, unless otherwise stated, for comparison.

6.2.1 No Pass Go

No Pass Go is played as per Go, except that players mayn't pass. This means that players may be forced to make *cold* moves[7] that result in the loss of material.

6.2.2 Ponnuki Go

Ponnuki Go (or Single Capture Go) is played as per the rules of Go, except that the first player to capture one of more pieces wins the game.

6.2.3 NoGo

NoGo is played as per the rules of Go, except that the first player with no legal moves loses [45].

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[5] http://www.cameronius.com/research/elegance

[6] "Mogal" stands for Modular Game Library.

[7] Moves that the player would prefer not to make.

6.2.4 Fill the Board

Fill the Board is a simple artificial described by Allis [40] as a counter-example for decision complexity, in which no move is better than any other. Players take turns placing a piece of their colour on empty cell, and the winner is the last player to move. The first player has a trivial win on odd boards and the second player on even boards, but note that the entire board must be filled to trigger the winning condition, despite all involved knowing who is going to win before a move is made.

6.2.5 Gomoku

Gomoku is played on a 15×15 Go board. Players take turns placing a piece of their colour and win by making five in a row of their colour, orthogonally or diagonally. The version implemented here does not use the standard opening rules or subsequent restrictions.

6.3 Reversi

Reversi is the classic piece-flipping game played on an 8×8 square board. It is a highly dynamic, with a constantly changing board state that can confuse even expert players until the last few moves.

6.3.1 Reversi+

Reversi+ is a custom variant of Reversi, played by the same rules except that line conversions are *chained*, so that any piece converted on a turn can trigger further conversions, which may trigger further conversions, etc. Reversi+ takes the dynamicism of Reversi to the extreme.

6.4 Hex

Hex is the quintessential *connection game* [19].It can be very deep on larger boards, and has the elegant property that exactly one player must win every game. The standard 11×11 board is implemented here.

6.4.1 Y

Y is a close relative of Hex played on a triangular board, on which players try to form a path of their colour between all three board sides. Again, exactly one player must win every game. Y may be described as a fundamental form of Hex; it is the simpler game on paper, but Hex is generally considered to be the deeper and clearer of the two in practice.

6.5 Tic Tac Toe

Tic Tac Toe is one of the simplest and most well-known games.It has only 576 unique board positions not counting rotations and reflections, and will always end in a draw except when played by beginners.

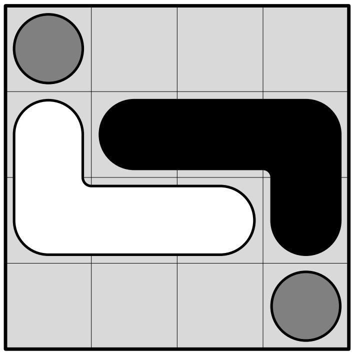

Fig. 1. The L Game starting position.

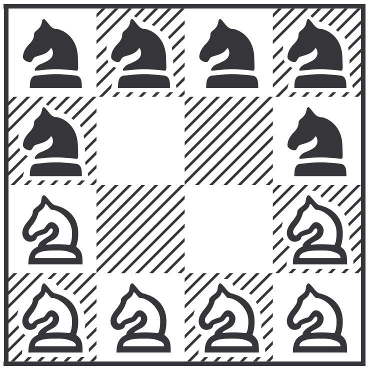

Fig. 2. Knight Panic starting position.

6.6 The L Game

The L Game is played on 4×4 square gird (Figure 1). Players take turns moving the L piece of their colour then optionally moving one of the neutral discs. A player loses if they cannot move their L piece. There are 2,296 possible positions, not counting rotations and reflections. Some care is required to avoid losing moves.

The L Game is often cited as an example of one of the simplest non-trivial games [46], and is also known for the dubious fact that games will continue without conclusion if played correctly. This could be seen as a flaw, but was apparently intended by its designer Edward de Bono to produce a meditative thought exercise that players could enjoy for as long as they wished. De Bono wanted to devise ”the simplest possible game that could be played with a high degree of skill” [47], which resonates strongly with the shibui ideal.

6.7 Chess

Chess is the standard strategy game for Western players, played on an 8×8 square board with 32 pieces of six different types. There are numerous terminating conditions, the most commonly achieved being to checkmate the enemy’s King. Chess and Go are renowned for their depth, as indicated by the number of players and number of skill levels in their well-established ranking schemes. The various piece types, special movement

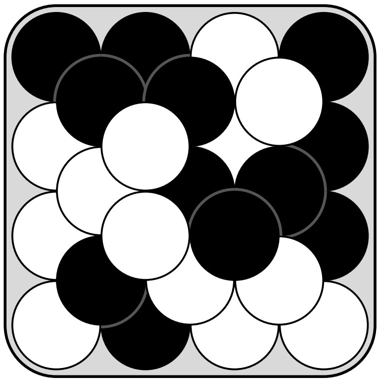

Fig. 3. A game of Spline won by White (size 2 line).

rules and winning condition make the rules of Chess relatively complex for a combinatorial game.

6.7.1 Knight Panic

Knight Panic is a puzzle-like game played on a 4×4 grid, that starts with six Chess Knights of each colour (Figure 2). Players take turns moving a Knight of their colour to an empty cell and win if the opponent has no legal moves. Games can go on ad infinitum, but it is difficult to achieve a losing position, so even less thought is required for planning moves than in the L Game.

6.8 Square Pyramidal (SP) Games

Square pyramidal (SP) games are played on a square grid of holes with balls that may stack upwards in a cannonball packing to form a pyramid. Pylos[8] is the most well known SP_4 game, while the *Shibumi set*[9] is an example of an SP_4 game system.

An advantage of SP games is that higher-level placement sites do not actually become playable until lower level pieces build up to support them, hence the number of playable sites can never exceed the base board size. This limits the branching factor to that of a 2D representation while allowing greater 3D state space complexity. The following two games are for the Shibumi set.

6.8.1 Spline

Spline[9] is played on the SP_4 board (Figure 3). Players take turns placing a ball of their colour at any playable site (board hole or 2×2 platform of support pieces) and win by making a *full line* of their colour, orthogonally or diagonally. A line is full if it extends the full width of the board at the current level, i.e. a line of four on the 4×4 level, a line of three on the 3×3 level, or a line of two on the 2×2 level. Every game must produce a winner before the last ball is played at the tip of the pyramid.

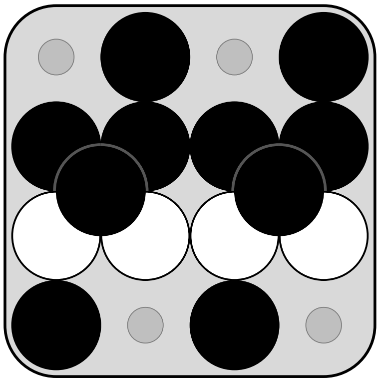

Fig. 4. A game of Spargo with White to play and win.

Rank	Elegance S_{eleg}		Shibui S_{shib}
	$+S_{eff}$	$-S_{eff}$	
#1	No Pass Go	No Pass Go	Spline
#2	NoGo	NoGo	No Pass Go
#3	Go (4×4)	Go (4×4)	NoGo
#4	Fill the Board	Fill the Board	Go (4×4)
#5	Tic Tac Toe	Ponnuki Go	Ponnuki Go
#6	Go (19×19)	Tic Tac Toe	Spargo
#7	Chess	Reversi	Reversi
#8	Knight Panic	Reversi+	The L Game
#9	Reversi	Spargo	Reversi+
#10	The L Game	Y	Chess
#11	Reversi+	Spline	Tic Tac Toe
#12	Spargo	Knight Panic	Y
#13	Ponnuki Go	Go (9×9)	Hex
#14	Y	The L Game	Gomoku
#15	Spline	Gomoku	Knight Panic
#16	Go (9×9)	Chess	Go (9×9)
#17	Hex	Hex	Go (19×19)
#18	Gomoku	Go (19×19)	Fill the Board

TABLE 1
Games ranked by elegance and shibui estimates.

6.8.2 Spargo

Spargo[9] is a 3D Go variant played on the SP_4 board, in which captured balls that support opponent's balls are not removed and remain active in the game as *zombies*. Figure 4 shows a position with White to move that hints at the depth of Spargo. White actually has a winning play here [10] despite despite being outnumbered by more than 2:1 and seemingly overwhelmed by a strong black group with two eyes. This result is very counterintuitive and surprising for a game of this size.

6.9 Results

Table 1 shows the example games ranked by their estimated elegance and shibui scores. Where rankings are tied, the average of their component scores is used to break the tie. Table 2 shows the component scores and resulting elegance S_{eleg} and shibui S_{shib} estimates.

We reiterate that these rankings are *not* a reflection on the quality of each game. For example, Go (19×19) is indisputably one of the world's great games, but does

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[8] Published by Gigamic.

[9] Recently published by Nestorgames.

[10] Proven at http://www.cameronius.com/games/spargo.

Game	C_{rul}	C_{eqp}	S_{simp}	M_{sep}	C_{mut}	S_{clar}	U_{brd}	U_{pc}	S_{eff}	C_{gt}	C_{ss}	S_{drel}	S_{eleg}	S_{shib}
Go (19×19)	0.452	0.881	**0.334**	0.000	0.000	**0.000**	0.998	1.000	**0.999**	1.000	1.000	**0.500**	**0.999**	**0.000**
Go (9×9)	0.452	0.536	**0.506**	0.000	0.046	**0.000**	0.915	0.630	**0.772**	1.000	0.760	**0.658**	**0.772**	**0.000**
Go (4×4)	0.452	0.204	**0.672**	0.722	0.039	**0.722**	0.963	1.000	**0.982**	0.390	0.130	**1.000**	**1.000**	**0.672**
No Pass Go	0.423	0.204	**0.687**	0.755	0.074	**0.755**	0.998	1.000	**0.999**	0.387	0.130	**1.000**	**1.000**	**0.687**
Ponnuki Go	0.452	0.204	**0.672**	0.806	0.033	**0.806**	0.999	0.803	**0.901**	0.249	0.130	**0.960**	**0.960**	**0.672**
NoGo	0.423	0.204	**0.687**	0.780	0.036	**0.780**	0.999	0.947	**0.973**	0.262	0.130	**1.000**	**1.000**	**0.687**
Fill the Board	0.000	0.204	**0.898**	0.000	0.000	**0.000**	1.000	1.000	**1.000**	0.266	0.130	**1.000**	**1.000**	**0.000**
Gomoku	0.151	0.778	**0.536**	0.025	0.000	**0.025**	0.999	0.243	**0.621**	1.000	1.000	**0.500**	**0.536**	**0.025**
Reversi	0.389	0.505	**0.553**	0.568	0.379	**0.568**	1.000	0.938	**0.969**	1.000	0.560	**0.893**	**0.969**	**0.553**
Reversi+	0.423	0.505	**0.536**	0.597	0.558	**0.442**	0.999	0.927	**0.963**	1.000	0.560	**0.893**	**0.963**	**0.442**
Hex	0.301	0.644	**0.528**	0.029	0.000	**0.029**	0.997	0.272	**0.636**	1.000	1.000	**0.500**	**0.636**	**0.029**
Y	0.151	0.512	**0.669**	0.036	0.000	**0.036**	0.998	0.203	**0.601**	0.982	0.600	**0.818**	**0.818**	**0.036**
Tic Tac Toe	0.151	0.079	**0.885**	0.075	0.000	**0.075**	1.000	1.000	**1.000**	0.111	0.060	**0.927**	**1.000**	**0.075**
The L Game	0.557	0.204	**0.620**	0.596	0.388	**0.596**	0.981	0.950	**0.965**	0.073	0.067	**0.546**	**0.965**	**0.546**
Chess	0.790	0.623	**0.294**	0.286	0.302	**0.286**	0.996	1.000	**0.998**	1.000	0.940	**0.532**	**0.998**	**0.286**
Knight Panic	0.389	0.174	**0.719**	0.005	0.301	**0.005**	0.989	0.954	**0.971**	0.068	0.130	**0.519**	**0.971**	**0.005**
Spline	0.151	0.277	**0.786**	0.754	0.000	**0.754**	0.998	0.605	**0.802**	0.371	0.263	**0.705**	**0.802**	**0.705**
Spargo	0.500	0.277	**0.612**	0.689	0.081	**0.689**	0.989	0.932	**0.961**	0.454	0.263	**0.864**	**0.961**	**0.612**

TABLE 2
Observed scores for example games.

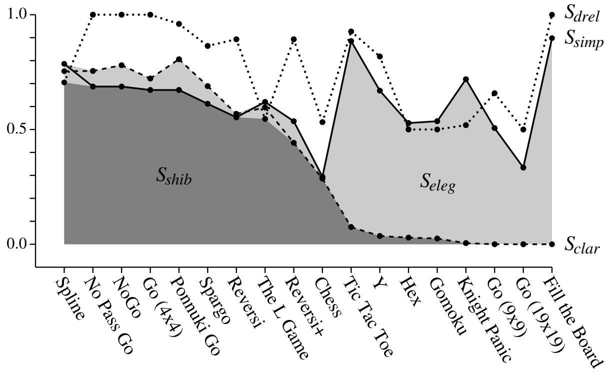

Fig. 5. Plot of key aesthetic scores, ranked by S_{shib}.

not fare well on the elegance or shibumi rankings due to its unusually large board size, number of pieces, and daunting lack of clarity for beginners.

The elegance rankings S_{eleg} in Table 1 are divided into two sub-columns:

- +S_{eff} rankings that include the S_{eff} contribution.
- –S_{eff} rankings that exclude the S_{eff} contribution.

This distinction is made to highlight the volatility of the elegance rankings, and the dominance of the S_{eff} component in determining the final elegance score. By contrast, the shibui rankings remain unchanged regardless of whether S_{eff} is included or not.

Fig. 5 shows a plot of the key aesthetic score estimates, ranked in order of final shibui score S_{shib}. The dark grey region indicates the area bounded by the S_{shib} estimates, while the light grey region shows the area bounded by the S_{eleg} estimates (excluding S_{eff}). Recall that S_{drel} does not contribute to elegance.

6.10 Discussion

It can be seen that No Pass Go, NoGo and Go (4×4) rank highly in all categories. This is not surprising as these are games with simple rules and equipment that provide more contest than might be expected. Recall that a high shibui score does not necessarily indicate the depth of a game, but the relative depth compared to that which might be expected from its outward form. The ranking of Spline as the most shibui example game is not a surprise, as it was specifically designed with the principles of shibui in mind.

The low elegance ranking of Hex is a surprise, as most players would describe Hex as a very elegant game. This elegance lies in the simplicity of its rules and the harmony between the rules and board topology, which is offset by the need for a large board and many pieces to make the game really work. The fact that the system did not pick up on the former may be due to the model's lack of an accurate measure of rule complexity, or may be due to the (unoptimised) GGP making suboptimal moves that do not truly reflect how actual games would unfold between human players.

The movement of Go (19×19) in the elegance rankings in the absence of S_{eff} from #6 to #18 may also be due to a quirk of the GGP, as it plays a weak game of Go and naïvely follows games to their conclusion rather than stopping when the result is obvious, using more pieces and board area than human experts would in practice.

The efficiency estimate S_{eff} appears to be an unreliable indicator, at least as measured here. A more sophisticated measurement might incorporate not only board and piece usage but influence over other components, in addition to some measure of functional rule efficiency.

The three deepest games – Chess, Hex and Go (19×19) – also have the lowest elegance estimates S_{eleg}, reinforcing that elegance is independent from quality. However, there are no real surprises in the shibui rankings. The shibui estimate S_{shib} appears to be successful in rewarding simpler games with non-trivial play, while penalising games with complex form and/or relatively little depth.

The aesthetic score plot shown in Fig. 5 reveals further insight into the measurements. It can be seen that S_{clar} is rather pessimistic, and in fact constrains the final S_{shib} score for most of the example games, while the S_{drel} score is if anything rather optimistic. Future work might involve investigating alternative methods for measuring these criteria that produce a more even spread across their range.

Two striking discrepancies in the graph are Tic Tac Toe and Fill the Board, which both have S_{clar} scores at the lower extreme but S_{simp} and S_{drel} scores at the upper extreme. There can be no argument that high S_{simp} scores are appropriate for these two extremely simple games. The high S_{drel} scores may be surprising as neither of these games could be called "deep" in any meaningful sense, but bear in mind that S_{drel} measures *relative* depth and not absolute depth, that is, S_{drel} does not indicate the actual depth of a game so much as its expected potential for depth given its observed complexity. This potential depth captured by S_{drel} is not realised for either of these games during play, but luckily the S_{clar} measurement compensates by capturing the fact no move is better than any other during play; all moves lead to a draw in Tic Tac Toe with trivial planning and a pre-defined result in Fill the Board with no planning at all. So on balance, these two metrics work hand-in-hand, just as depth and clarity work hand-in-hand in actual practice.

6.11 Practical Uses

So what are the real world applications of these aesthetic measurements of elegance and shibui? They represent new axes along which games may be measured, largely independent of existing classifications, which may be used to:

- Direct the automated search for elegant and/or shibui games.
- Facilitate machine assistance for players and designers for quickly detecting elegant and/or shibui designs.
- Match appropriate games with casual players (elegance) and hardcore players (shibui).
- Indicate suitable games for players pressed for time, that are easy to learn and play quickly (elegance), but which still offer sufficient challenge (shibui).
- Indicate games with simpler rule sets, for younger players or players with learning disabilities who have trouble remembering rule sets.
- Suggest appropriate games for players more attuned to the aesthetics of the *ludic* (game-play) experience rather than the competitive challenge.
- Indicate new games with simpler equipment but deeper play (shibui), for budget-conscious game publishing houses.

7 CONCLUSION

This paper introduces notions of elegance and shibui in combinatorial game design. Player-centric computational models, based on known principles of game design, are developed in order to produce estimates of these values for given games. While elegance estimates proved somewhat unreliable on preliminary tests, shibui estimates look promising as a potential new dimension for evaluating and classifying games.

The main contributions of this paper are: 1) to define elegance and shibui in terms of combinatorial games, 2) to lay the theoretical foundations for relevant computational models, and 3) to demonstrate that such ephemeral concepts *can* actually be estimated mathematically. Future work might involve user surveys to verify the results against actual opinion from game players (both casual and hardcore) and designers. However, this could be a difficult task as opinion on these matters varies widely from player to player depending on their background, motivation and knowledge of a given game; the obtained data would likely be less cohesive than that obtained for similar surveys on game design [15].

The metrics proposed in this paper are founded on known principles of combinatorial game design, but are only estimates and should be taken with a grain of salt, and may well be superseded by more sophisticated and accurate measurements in future. They assume simple linear relationships among the measured attributes, and might benefit from more complex nonlinear mappings.

It is argued that the elegance of a game is largely independent of its quality, but may still be useful for identifying games of particular character. Shibui, however, resonates more strongly with accepted principles of good game design, and is potentially more useful as an indicator of both elegance and quality in games, epitomising the "minute to learn, lifetime to master" ideal of combinatorial game design.

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